

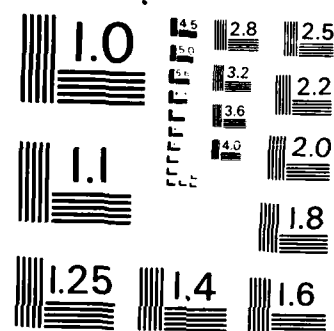
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## REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b RESTRICTIVE MARKINGS	
2a SECURITY CLASSIFICATION AUTHORITY		3 DISTRIBUTION AVAILABILITY OF REPORT 1. <input checked="" type="checkbox"/> Approved for public release 2. <input type="checkbox"/> Distribution unlimited	
4 DECLASSIFICATION/DOWNGRADING SCHEDULE		5 MONITORING ORGANIZATION REPORT NUMBER(S) <b>AFOSR-TR-85-0236</b>	
4 PERFORMING ORGANIZATION REPORT NUMBER(S)			
1a NAME OF PERFORMING ORGANIZATION University of California Santa Barbara		6b OFFICE SYMBOL (If applicable)	
7a ADDRESS (City, State, and ZIP Code) Santa Barbara, CA 93106		7a NAME OF MONITORING ORGANIZATION	
NAME OF FUNDING/SPONSORING ORGANIZATION		7b ADDRESS (City, State, and ZIP Code) Department of Mathematics, University of Maryland, College Park, MD 20742	
8a OFFICE SYMBOL (If applicable) N/A		9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-83-0150	
ADDRESS (City, State, and ZIP Code) Building 402 DC 20337		10 SOURCE OF FUNDING NUMBERS	
		PROGRAM ELEMENT NO 61102F	PROJECT NO 2304
		TASK NO A3	WORK UNIT ACCESSION NO
TITLE (Include Security Classification) STABILITY ANALYSIS OF FINITE DIFFERENCE SCHEMES FOR HYPERBOLIC SYSTEMS, AND PROBLEMS IN APPLIED AND COMPUTATIONAL LINEAR ALGEBRA			
12 PERSONAL AUTHOR(S) MARCUS, MARVIN AND GOLDBERG, MOSHE			
13a TYPE OF REPORT INTERIM ANALYSIS	13b TIME COVERED FROM 050184 TO 043085	14 DATE OF REPORT (Year, Month, Day) 1985, AUGUST, 22	15 PAGE COUNT 27
16 SUPPLEMENTARY NOTATION			
17 COSATI CODES		18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	
19 ABSTRACT (Continue on reverse if necessary and identify by block number)			
<p>The Air Force supported research of M. Goldberg during the period May 1, 1984 - April 30, 1985, consists of two main achievements: (a) New stability criteria for finite difference approximations to initial-boundary value problems associated with the parabolic system</p> $\partial u(x,t) / \partial t = A \partial u(x,t) / \partial x + B u(x,t) + f(x,t), \quad x \geq 0, t \geq 0.$ <p>(b) A study of submultiplicativity properties for arbitrary matrix norms, and a generalization of these properties for the well-known <math>\ell_p</math> norms on matrices.</p> <p style="text-align: right;">over</p>			
20 DISTRIBUTION AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS		21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED	
22a NAME OF RESPONSIBLE INDIVIDUAL David W. Fox		22b TELEPHONE (Include Area Code) 202 767- 5026	22c OFFICE SYMBOL N/A

DTIC FILE COPY

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The research of M. Marcus falls in the following categories:

(1) The relationship between the algebraic properties of a finite complex matrix and the geometric properties of its numerical range; (2) the eigenvalue containment properties of the numerical range and their use to obtain computationally significant estimates of such associated numbers as the condition number, the parameters in Tchebychev iteration for an  $n$ -square real linear system and initial estimates in various iterative eigenvalue determination procedures; (3) the foundations of a theory for the numerical range of certain operators on various symmetry classes of tensors, e.g., the Grassmann and completely symmetric spaces. A typical instance of such results are the classical inequalities of H. Weyl relating eigenvalues and singular values.

AFOSR-TR. 85-0936

INTERIM SCIENTIFIC REPORT

Stability Analysis of Finite Difference Schemes for  
Hyperbolic Systems, and Problems in Applied and  
Computational Linear Algebra

Grant AFOSR-83-0150

Period: 1 May 1984 - 30 April 1985

Principal Investigators: Marvin Marcus  
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RECEIVED  
 OFFICE OF THE SECRETARY  
 AIR FORCE  
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 This is a final report  
 from the  
 Chief, Operations Information Division

STABILITY ANALYSIS OF FINITE DIFFERENCE SCHEMES FOR  
HYPERBOLIC SYSTEMS, AND PROBLEMS IN APPLIED AND  
COMPUTATIONAL LINEAR ALGEBRA

SUMMARY

This interim report describes the following two projects carried out under <sup>an</sup> Air Force grant AFOSR-83-0150, during the period May 1, 1984 - April 30, 1985: (a) Stability criteria for difference approximations to hyperbolic systems, and multiplicativity of matrix norms, by M. Goldberg; (b) Problems in applied and computational linear algebra, by M. Marcus. The aim of these projects was to achieve better understanding of useful computational techniques for hyperbolic initial-boundary value problems, and to improve basic mathematical tools often used in numerical analysis and applied mathematics.

STABILITY CRITERIA FOR DIFFERENCE APPROXIMATIONS TO  
HYPERBOLIC SYSTEMS, AND MULTIPLICATIVITY OF MATRIX NORMS

Moshe Goldberg

ABSTRACT

The Air Force supported research of M. Goldberg during the period May 1, 1984 - April 30, 1985, consists of two main achievements: (a) New stability criteria for finite difference approximations to initial-boundary value problems associated with the hyperbolic system

$$\partial u(x,t)/\partial t = A\partial u(x,t)/\partial x + Bu(x,t) + f(x,t), \quad x \geq 0, \quad t \geq 0.$$

(b) A study of submultiplicativity properties for arbitrary matrix norms, and a complete description of those properties for the well-known  $l_p$  norms on matrices.



STABILITY CRITERIA FOR DIFFERENCE APPROXIMATIONS  
TO HYPERBOLIC SYSTEMS, MULTIPLICATIVITY OF MATRIX NORMS

Moshe Goldberg

1. Convenient Stability Criteria for Difference Approximations  
to Hyperbolic Initial-Boundary Value Problems.

Consider the first order system of hyperbolic partial  
differential equations

$$\partial u(x,t)/\partial t = A \partial u(x,t)/\partial x + Bu(x,t) + f(x,t), \quad x \geq 0, \quad t \geq 0,$$

where  $u(x,t)$  is the unknown vector;  $A$  a Hermitian matrix  
of the form  $A = A_1 \oplus A_2$ , where  $A_1$  is negative definite and  
 $A_2$  is positive definite; and  $f(x,t)$  is a given vector. The  
problem is well posed in  $L_2(0,\infty)$  if initial values

$$u(x,t) = u^0(x), \quad L_2(0,\infty), \quad x \geq 0,$$

and boundary conditions

$$u_1(0,t) = Su_2(0,t) + g(t), \quad t \geq 0,$$

are prescribed. Here  $u_1$  and  $u_2$  are the inflow and out-  
flow parts of  $u$  corresponding to the partition of  $A$ , and  
 $S$  is a coupling matrix.

In the past summer, E. Tadmor and I [19] have revised and extended our previous paper [18] in which we obtained new, easily checkable stability criteria for a wide class of finite difference approximations for the above initial-boundary value problem. Our difference approximations consist of a general difference scheme -- explicit or implicit, dissipative or not, two-level or multi-level -- and boundary conditions of a rather general type.

In our work we restricted attention to the case where the outflow boundary conditions are translatable, i.e., determined at all boundary points by the same coefficients. This, however, is not a severe limitation since such boundary conditions are commonly used in practice. In particular, when the numerical boundary consists of a single point, the boundary conditions are translatable by definition.

Throughout our papers [18, 19] we assumed that the basic scheme is stable for the pure Cauchy problem, and that the assumptions which guarantee the validity of the stability theory of Gustafsson, Kreiss and Sundstrom [20] hold. With this in mind we raised the question of stability for the entire difference approximation.

The first step in our stability analysis was to prove that the approximation is stable if and only if the scalar outflow components of its principal parts are stable. This reduces our global stability question to that of a scalar homogeneous outflow problem of the form

$$\partial u / \partial t = a \partial u / \partial x, \quad a = \text{constant} > 0, \quad x \geq 0, \quad t \geq 0$$

$$u(x, 0) = u^0(x), \quad x \geq 0; \quad u(0, t) = 0, \quad t \geq 0.$$

The stability criteria obtained in [18, 19] for the reduced problem depend both on the basic difference scheme and on the boundary conditions, but very little on the interaction between the two. Such criteria eliminate the need to analyze the intricate and often complicated interaction between the basic scheme and the boundary conditions; hence providing in many cases a convenient alternative to the well known stability criteria of Kreiss [24] and of Gustafsson, Kreiss and Sundstrom [20]. It should be pointed out that our old scheme-independent stability criteria in [15, 16] easily follow from the present criteria in [18, 19].

Having the new criteria in [18, 19], we reestablished all the examples in our previous papers [15, 16]. We showed, for instance, that if the basic scheme is arbitrary (dissipative or not) and the boundary conditions are generated by either the explicit or implicit right-sided Euler schemes, then overall stability is assured. For a dissipative basic scheme we proved stability if the boundary conditions are determined by either oblique extrapolation, the Box-scheme, or the right-sided weighted Euler scheme. These and other examples incorporate most of the cases discussed in recent literature [3, 4, 15, 16, 20, 21, 23, 25, 30, 33, 34, 36].

We treated in [18, 19] some new examples as well. Among these we found that if the basic scheme is arbitrary and two-level, then horizontal extrapolation at the boundary maintains overall stability. Other stable cases occur when the basic scheme is given by either the backward (implicit) Euler scheme or by the Crank-Nicholson scheme, and the boundary conditions are determined by oblique extrapolation. Such examples, where neither the basic scheme nor the boundary conditions are necessarily dissipative, could not have been handled by our previous results in [15, 16].

We drew great satisfaction from the fact that our theory and examples in [18, 19] were used already by a number of authors, including Berger [2], LeVeque [27], South and Hafez [35], Thuné [37], Trefethen [38, 39], and Yee [41]. Thuné [37], in his effort to provide a software package for stability analysis of finite difference approximations to hyperbolic initial-boundary value problems, says for example: "...Another approach has been to derive new criteria, based on the Gustafsson-Kreiss-Sundstrom theory but more convenient for practical use... The most far-reaching work along these lines has been made by Goldberg and Tadmor [15, 16, 18]..."

## 2. Submultiplicativity and Mixed Submultiplicativity of Matrix Norms.

During 1980-1982, E.G. Straus (now deceased) and I [11, 12] investigated submultiplicativity properties of norms and

seminorms on operator algebras -- an important subject in many fields of numerical analysis and applied mathematics. In our work we study an arbitrary normed vector space  $V$  over the complex field  $C$ , with an algebra  $L(V)$  of linear bounded operators on  $V$ , and a seminorm  $N$  on  $L(V)$ . If  $N$  is positive definite, i.e.,  $N(A) > 0$  for all  $A \neq 0$ , then we call  $N$  an operator norm. If in addition,  $N$  is submultiplicative, namely  $N(AB) \leq N(A)N(B)$  for all  $A, B \in L(V)$ , then  $N$  is called an operator norm on  $L(V)$ .

Given a seminorm  $N$  on  $L(V)$  and a fixed constant  $\mu > 0$ , then obviously  $N_\mu \equiv \mu N$  is a seminorm too. Similarly,  $N_\mu$  is a norm if and only if  $N$  is. In both cases,  $N_\mu$  may or may not be submultiplicative. If it is, we say that  $\mu$  is a multiplicativity factor for  $N$ .

Having these definitions we proved the following in [11]:

- (i) If  $N$  is a norm or a nontrivial seminorm on  $L(V)$ , then  $N$  has multiplicativity factors if and only if

$$\mu_N \equiv \sup\{N(AB) : N(A) = N(B) = 1\} < \infty.$$

- (ii) If  $\mu_N < \infty$ , then  $\mu$  is a multiplicativity factor for  $N$  if and only if  $\mu \geq \mu_N$ .

Special attention was given by us to the finite dimensional case where it suffices, of course, to consider  $C_{n \times n}$ , the algebra of  $n \times n$  complex matrices. We proved in this case that while all norms on  $C_{n \times n}$  do have multiplicativity

factors, indefinite seminorms on  $C_{n \times n}$  never have such factors. In the infinite dimensional case, however, the situation is less decisive, i.e., there exist norms and nontrivial seminorms on  $L(V)$  which may or may not have multiplicativity factors.

In both the finite and infinite-dimensional cases we proved that if  $M$  and  $N$  are seminorms on  $L(V)$  such that  $M$  is multiplicative, and if  $\eta \geq \xi > 0$  are constants satisfying

$$\xi M(A) < N(A) < \eta M(A) \quad \text{for all } A \in L(V),$$

then any  $\mu$  with  $\mu \geq \eta/\xi^2$  is a multiplicativity factor for  $N$ .

Using this result we showed, for example, that if  $V$  is an arbitrary Hilbert space and

$$r(A) = \sup\{|(Ax, x)| : x \in V, |x| = 1\}, \quad A \in L(V),$$

is the classical numerical radius, then  $\mu r$  is an operator norm if and only if  $\mu \geq 4$ ; thus, in particular finding that  $4r$  is an operator norm. This assertion is of interest since the numerical radius  $r$  is one of the best known non-multiplicative norms [1, 5, 17, 22, 32], and it plays an important role in stability analysis of finite difference schemes for multi-space-dimensional hyperbolic initial-value problems [17, 26, 28, 40]. Similar results for generalized numerical radii are given in [9-12, 14].

In our last joint effort, Straus and I [13] obtained multiplicativity factors for the well known  $\ell_p$  norms ( $1 \leq p \leq \infty$ ):

$$|A|_p = \{\sum_{ij} |a_{ij}|^p\}^{1/p}, \quad A = (a_{ij}) \in C_{n \times n}.$$

It was shown by Ostrowski [31] that these norms are multiplicative if and only if  $1 \leq p \leq 2$ . For  $p \geq 2$  we have shown in [13] that  $\mu$  is a multiplicativity factor for  $|A|_p$  if and only if  $\mu \geq n^{1-2/p}$ ; hence, obtaining the useful result that  $n^{1-2/p}|A|_p$  is a submultiplicative norm on  $C_{n \times n}$ .

One of my objectives in 1984 was to extend the above ideas to mixed submultiplicativity. More precisely, let

$$N_1 : C_{m \times n} \rightarrow R, \quad N_2 : C_{m \times k} \rightarrow R, \quad N_3 : C_{k \times n} \rightarrow R$$

be given norms on the class of  $m \times n$ ,  $m \times k$ , and  $k \times n$  complex matrices, respectively. Unless  $m = n$ ,  $C_{m \times n}$  is not an algebra; so the question whether  $N_1$  is submultiplicative is irrelevant. Instead, it seems natural to ask whether there exist constants  $\mu > 0$  for which

$$N_1(AB) \leq \mu N_2(A)N_3(B) \quad \text{for all } A \in C_{m \times k}, B \in C_{k \times n}. \quad (1)$$

We call a constant  $\mu > 0$  which satisfies (1), a multiplicativity factor for  $N_1$  with respect to  $N_2$  and  $N_3$ . In

particular, if (1) holds with  $\mu = 1$ , we say that  $N_1$  is submultiplicative with respect to  $N_2$  and  $N_3$ .

With these definitions, we can prove [8]:

- (i)  $N_1$  has multiplicativity factors with respect to  $N_2$  and  $N_3$ .
- (ii)  $\mu > 0$  is a multiplicativity factor for  $N_1$  with respect to  $N_2$  and  $N_3$  if and only if

$$\mu \geq \mu_{\min} \equiv \max\{N_1(AB) : A \in C_{m \times k}, B \in C_{k \times n}, N_2(A) = N_3(B) = 1\}.$$

In view of this theorem, I revisited the above mentioned  $\ell_p$  norms for matrices, and determined explicitly, for arbitrary  $p, q, r$  such that  $1 \leq p, q, r \leq \infty$ , the best (least) constant  $\mu_{\min}$  for which

$$|AB|_p \leq \mu_{\min} |A|_q |B|_r \quad \text{for all } A \in C_{m \times k}, B \in C_{k \times n}.$$

The desired constant, [8], is

$$\mu_{\min} = \lambda_{pq}^{(m)} \lambda_{pr}^{(n)} \lambda_{q'r}^{(k)},$$

where  $1/r + 1/r' = 1$ , and

$$\lambda_{pq}^{(m)} \equiv \begin{cases} 1, & p \geq q \\ m^{1/p - 1/q}, & q \geq p. \end{cases}$$

This extends some partial results in [6, 7].



## REFERENCES

1. C.A. Berger, On the numerical range of powers of an operator, Not. Amer. Math. Soc. 12 (1965), Abstract No. 625-152.
2. M.J. Berger, Stability of Interfaces with Mesh Refinement, Report No. 83-42, 1983, ICASE, NASA Langley Research Center, Hampton, Virginia.
3. M. Goldberg, On a boundary extrapolation theorem by Kreiss, Math. Comp. 31 (1977), 469-477.
4. M. Goldberg, On boundary extrapolation and dissipative schemes for hyperbolic problems, Proceedings of the 1977 Army Numerical Analysis and Computer Conference, ARO Report 77-3, 157-164.
5. M. Goldberg, On certain finite dimensional numerical ranges and numerical radii, Linear and Multilinear Algebra 7 (1979), 329-342.
6. M. Goldberg, Some inequalities for  $\ell_p$  norms of matrices, in "General Inequalities 4", edited by W. Walter, Birkhäuser Verlag, Basel, 1984, pp. 185-189.
7. M. Goldberg, Multiplicativity of  $\ell_p$  norms for matrices. II, Linear Algebra Appl. 62 (1984), 1-10.
8. M. Goldberg, Mixed multiplicativity and  $\ell_p$  norms for matrices, Linear Algebra Appl., accepted.
9. M. Goldberg and E.G. Straus, Norm properties of C-numerical radii, Linear Algebra Appl. 24 (1979), 113-131.
10. M. Goldberg and E.G. Straus, Combinatorial inequalities, matrix norms, and generalized numerical radii, in "General Inequalities 2", edited by E.F. Beckenbach, Birkhäuser Verlag, Basel, 1980, 37-46.
11. M. Goldberg and E.G. Straus, Operator norms, multiplicativity factors, and C-numerical radii, Linear Algebra Appl. 43 (1982), 137-159.
12. M. Goldberg and E.G. Straus, Combinatorial inequalities, matrix norms, and generalized numerical radii. II, in "Generalized Inequalities 3", edited by E.F. Beckenbach, Birkhäuser Verlag, Basel, 1983, 195-204.

13. M. Goldberg and E.G. Straus, Multiplicativity of  $\ell_p$  norms for matrices, Linear Algebra Appl. 52-53 (1983), 351-360.
14. M. Goldberg and E.G. Straus, Multiplicativity factors for C-numerical radii, Linear Algebra Appl. 54 (1983), 1-16.
15. M. Goldberg and E. Tadmor, Scheme-independent stability criteria for difference approximations of hyperbolic initial-boundary value problems. I, Math. Comp. 32 (1978), 1097-1107.
16. M. Goldberg and E. Tadmor, Scheme-independent stability criteria for difference approximations of hyperbolic initial-boundary value problems. II, Math. Comp. 36 (1981), 605-626.
17. M. Goldberg and E. Tadmor, On the numerical radius and its applications, Linear Algebra Appl. 42 (1982), 263-284.
18. M. Goldberg and E. Tadmor, New stability criteria for difference approximations of hyperbolic initial-boundary value problems, in "Large Scale Computations in Fluid Mechanics", edited by B.E. Engquist et al., American Mathematical Society, Providence, Rhode Island, 1985, 177-192.
19. M. Goldberg and E. Tadmor, Convenient stability criteria for difference approximations of hyperbolic initial-boundary value problems, Math. Comp. 44 (1985), 361-377.
20. B. Gustafsson, H.-O. Kreiss and A. Sundstrom, Stability theory of difference approximations for mixed initial boundary value problems. II, Math. Comp. 26 (1972), 649-686.
21. B. Gustafsson and J. Oliger, Stable boundary approximations for implicit time discretizations for gas dynamics, SIAM J. Sci. Stat. Comput. 3 (1982), 408-421.
22. P.R. Halmos, "A Hilbert Space Problem Book", Van Nostrand, New York, 1967.
23. H.-O. Kreiss, Difference approximations for hyperbolic differential equations, in "Numerical Solution of Partial Differential Equations", edited by J.H. Bramble, Academic Press, New York, 1966, 51-58.
24. H.-O. Kreiss, Stability theory for difference approximations of mixed initial-boundary value problems. I, Math. Comp. 22 (1968), 703-714.
25. H.-O. Kreiss and J. Oliger, "Methods for the Approximate Solution of Time Dependent Problems", GARP Publication Series No. 10, Geneva, 1973.

26. P.D. Lax and B. Wendroff, Difference schemes for hyperbolic equations with high order of accuracy, Comm. Pure Appl. Math. 17 (1964), 381-391.
27. R.J. LeVeque, Intermediate Boundary conditions for Time-Split Methods Applied to Hyperbolic Partial Differential Equations, preprint, Courant Institute of Mathematical Sciences, 1983.
28. A. Livne, Seven point difference schemes for hyperbolic equations, Math. Comp. 29 (1975), 425-433.
29. M. Marcus and M. Sandy, Three elementary proofs of the Goldberg-Straus theorem on numerical radii, Linear and Multilinear Algebra 11 (1982), 243-252.
30. S. Osher, Systems of difference equations with general homogeneous boundary conditions, Trans. Amer. Math. Soc. 137 (1969), 177-201.
31. A.M. Ostrowski, Über Normen von Matrizen, Math. Z. 63 (1955), 2-18.
32. C. Pearcy, An elementary proof of the power inequality for the numerical radius, Mich. Math. J. 13 (1966), 289-291.
33. G. Skolermo, How the Boundary Conditions Affect the Stability and Accuracy of Some implicit Methods for Hyperbolic Equations, Report No. 62., 1975, Dept. of Computer Science, Uppsala University, Uppsala, Sweden.
34. G. Skolermo, Error Analysis for the Mixed Initial Boundary Value Problem for Hyperbolic Equations, Report No. 63, 1975, Dept. of Computer Science, Uppsala University, Uppsala, Sweden.
35. J.C. South, Jr. and M.M. Hafez, Stability analysis of intermediate boundary conditions in approximate factorization schemes, to appear.
36. E. Tadmor, Scheme-Independent Stability Criteria for Difference Approximations to Hyperbolic Initial-Boundary Value Systems, Ph.D. Thesis, Department of Mathematical Sciences, Tel Aviv University, Tel Aviv, Israel, 1978.
37. M. Thuné, IBSTAB- A Software System for Automatic Stability Analysis of Difference Methods for Hyperbolic Initial-Boundary Value Problems, Report No. 93, 1984, Department of Computer Science, Uppsala University, Uppsala, Sweden.
38. L.N. Trefethen, Wave Propagation and Stability for Finite Difference Schemes, Ph.D. Thesis, Report No. STAN-CS-82-905, Computer Science Department, Stanford University, Stanford, California, 1982.

39. L.N. Trefethen, Stability of Finite Difference Models Containing Two Boundaries or Interfaces, Report No. 84-11, 1984, ICASE, NASA Langley Research Center, Hampton, Virginia.
40. E. Turkel, Symmetric hyperbolic difference schemes and matrix problems, Linear Algebra Appl. 16 (1977), 109-129.
41. H.C. Yee, Numerical Approximation of Boundary Conditions with Applications to Inviscid Equations of Gas Dynamics, NASA Technical Memorandum 81265, 1981, NASA Ames Research Center, Moffett Field, California.

## PUBLICATIONS

Moshe Goldberg

May 1, 1984 - April 30, 1985

1. Some inequalities for  $\ell_p$  norms of matrices, in "General Inequalities 4", edited by W. Walter, Birkhauser Verlag, Basel, 1984, 185-189.
2. Multiplicativity of  $\ell_p$  norms for matrices. II, Linear Algebra and Its Applications 62 (1984), 1-10.
3. New stability criteria for difference approximations of hyperbolic initial-boundary value problems (with E. Tadmor), in "Large-Scale Computations in Fluid Mechanics", edited by B.E. Engquist, S.J. Osher, and R.C.J. Somerville, American Mathematical Society, Providence, Rhode Island, 1985, 177-192.
4. Ernst G. Straus (1922-1983), Linear Algebra and Its Applications 63 (1985), 1-19.
5. Convenient stability criteria for difference approximations of hyperbolic initial-boundary value problems (with E. Tadmor), Mathematics of Computation 44 (1985), 361-377.
6. Mixed multiplicativity and  $\ell_p$  norms for matrices, Linear Algebra and Its Applications, 1985, accepted.

## INTERACTIONS

Moshe Goldberg

May 1, 1984 - April 30, 1985

1. Invited Speaker, The 1984 Annual Meeting of the Israel Mathematical Union, Applied Mathematics Session, Tel Aviv University, Tel Aviv, Israel, May 1984, title: "Convenient stability criterial for difference approximations of hyperbolic initial-boundary value problems".
2. Speaker (two talks), The Gatlinburg IX Conference on Numerical Algebra, University of Waterloo, Waterloo, Canada, July 1984, titles: "Generalizations of the Perron-Frobenius Theorem", and "Norms and multiplicativity".
3. Invited Attendee, The U.S.-Israel Binational Workshop on the Impact of Supercomputers on the Next Decade of Computational Fluid Dyinamics, Jerusalem, December 1984, Panel Discussion.
4. Invited Speaker, the Haifa 1984 Matrix Theory Conference, University of Haifa and Technion-Israel Institute of Technology, Haifa, Israel, December 1984, title: "Submultiplicativity of matrix norms and operator norms".
5. Invited Speaker, Polytechnic Institute of New York, Brooklyn, New York, Departmental Seminar, September 1984, title: "On matrix norms and submultiplicativity".
6. Invited Speaker, Joint French-Israeli Mathematical Symposium on Linear and Nonlinear Partial Differential Equations, Numerical Analysis, and Geometry of Banach Spaces, The Israel Academy of Sciences and Humanities, Jerusalem, March 1985, title: "Stability criteria for difference approximations of hyperbolic initial-boundary value problems".

## PROBLEMS IN APPLIED AND COMPUTATIONAL LINEAR ALGEBRA

Marvin Marcus

### ABSTRACT

The research of M. Marcus falls in the following categories:

(1) The relationship between the algebraic properties of a finite complex matrix and the geometric properties of its numerical range; (2) the eigenvalue containment properties of the numerical range and their use to obtain computationally significant estimates of such associated numbers as the condition number, the parameters in Tchebychev iteration for an  $n$ -square real linear system and initial estimates in various iterative eigenvalue determination procedures; (3) the foundations of a theory for the numerical range of certain operators on various symmetry classes of tensors, e.g., the Grassmann and completely symmetric spaces. A typical instance of such results are the classical inequalities of H. Weyl relating eigenvalues and singular values.

# PROBLEMS IN APPLIED AND COMPUTATIONAL LINEAR ALGEBRA

Marvin Marcus

My Air Force supported research during the period May 1, 1984 - April 30, 1985, consists of the following papers:

1. Singular Values and Numerical Radii (to appear in Linear and Multilinear Algebra)

This paper proves the following result relating singular values and the numerical radius of an  $n$ -square complex matrix  $A$ . Let  $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n$  be the singular values of  $A$  and let  $r(A)$  denote the numerical radius of  $A$ . Then

$$(\alpha_1 + \alpha_2 + \cdots + \alpha_n)/n \leq r(A). \quad (1)$$

Moreover, equality holds in (1) if and only if  $A/r(A)$  is unitarily similar to the direct sum of a diagonal unitary matrix and unit multipliers of  $2 \times 2$  matrices of the form

$$\begin{bmatrix} 1 & d \\ -d & -1 \end{bmatrix}$$

where  $0 < |d| \leq 1$ .



We intend to generalize (1) to

$$E_k(\alpha_1, \dots, \alpha_n) / \binom{n}{k} \leq r_d^k(A) \quad (2)$$

where  $r_d^k$  is the  $k^{\text{th}}$  decomposable numerical radius of  $A$ :

$$r_d^k(A) = \max |\det[(Ax_i, x_j)]|$$

where the vectors  $x_1, \dots, x_k$  vary over all sets of  $k$  orthonormal vectors. (See M. Marcus and P. Andresen, Linear Algebra Appl. 16 (1977), pp. 131-151.) Determining the case of equality in (2) appears to be difficult.

## 2. Conditions for Generalized Numerical Range to be Real (to appear in Linear Algebra and Its Applications)

Let  $C$  and  $A$  be  $n$ -square complex matrices. The  $C$  numerical range of  $A$  is the set of numbers

$$W(C, A) = \{\text{tr}(CU^*AU) \mid U \text{ unitary}\}.$$

We prove in this paper that if  $W(C, A)$  is contained in a fixed line parallel to the real axis then at least one of  $C$  and  $A$  must be normal. We also characterize the  $A$  and  $C$  for which  $W(C, A)$  is a subset of the real line. We show that for the classical numerical range,  $W(CU^*AU)$  is a subset of the real line for all unitary  $U$  if and only if at least one of  $C$  and  $A$  is scalar and their product is hermitian. We are

interested in extending this type of result to functions of  $CU*AU$  other than the trace. Moreover, based on earlier work (Marcus and Filippenko), if  $W(CU*AU)$  is a subset of the unit disk, then suitable conditions on the eigenvalues of  $C$  or  $A$  (or both) will undoubtedly imply that  $C$  or  $A$  (or both) are closely related to unitary matrices.

3. Ryser's Permanent Identity in the Symmetric Algebra  
(to appear in Linear and Multilinear Algebra)

The polynomial algebra over a field is isomorphic to the symmetric algebra over a vector space. Using this isomorphism, several identities for the permanent function are derived; one of which is Ryser's expansion theorem. Nijenhuis and Wilf have published an efficient FORTRAN code for implementing the identity.

4. Construction of Orthonormal Bases in Higher Symmetry  
Classes of Tensors (to appear in Linear and Multilinear  
Algebra)

This paper presents a method for constructing an orthonormal basis for a symmetry class of tensors from an orthonormal basis of the underlying vector space. The basis so obtained is not composed of decomposable symmetrized tensors.

In fact, it is shown that for symmetry classes of tensors whose associated character has degree higher than one, it is impossible to construct an orthogonal basis of decomposable symmetrized tensors from any basis of the underlying vector space. The paper ends with an open question on the possibility of a symmetry class having an orthonormal basis of decomposable symmetrized tensors.

5. An Exponential Group (Linear and Multilinear Algebra 16 (1984), pp. 97-99)

This short note shows how certain binomial identities can be derived by using the exponential of a matrix.

## PUBLICATIONS

Marvin Marcus

May 1, 1984 - April 30, 1985

1. Discrete Mathematics: A Computational Approach, Computer Science Press Inc., Rockville, Maryland, pp. 1-329.
2. Bilinear ranges, convexity and elementary doubly stochastic matrices (with M. Sandy and K. Kidman), Linear and Multilinear Algebra, in press.
3. Products of doubly stochastic matrices (with K. Kidman and M. Sandy), Linear and Multilinear Algebra, Vol. 15 (1984), pp. 331-340.
4. Unitarily invariant generalized matrix norms and Hadamard products (with K. Kidman and M. Sandy), Linear and Multilinear Algebra, Vol. 16 (1984), pp. 197-213.
5. An exponential group, Linear and Multilinear Algebra, Vol. 16 (1984), pp. 97-99.
6. Conditions for the generalized numerical range to be real (with M. Sandy), Linear Algebra and Its Applications, to appear in August 1985.
7. Ryser's permanent identity in the symmetric algebra (with M. Sandy) to appear in Linear and Multilinear Algebra.

## INTERACTIONS

Marvin Marcus

May 1, 1984 - April 30, 1985

Extensive correspondence with researchers in linear algebra, matrix theory, convexity theory, and numerical algebra in various editorial and reviewing capacities:

1. Mathematics Editor, Computer Science Press
2. Editor, Linear and Multilinear Algebra, published by Gordon and Breach, Science Publishers, Inc.
3. Associate Editor, Linear Algebra and Its Applications, Elsevier Science Publishing Co., Inc.
4. Member of the Editorial Board, Pure and Applied Mathematics Series, Marcel Dekker, Inc.
5. Editor, Linear Algebra Volumes of Encyclopedia of Applicable Mathematics, Addison-Wesley Publishing Co.
6. Associate Editor, Advanced Problem Section, American Mathematical Monthly.
7. Referee and Reviewer for the following journals:
  - Linear and Multilinear Algebra
  - Linear Algebra and Its Applications
  - Duke Journal
  - Proceedings of the AMS
  - Transactions of the AMS
  - Bulletin of the AMS
  - Mathematical Reviews
  - Memoirs of the MAA

American Mathematical Monthly  
Canadian Journal of Mathematics  
Pacific Journal of Mathematics  
Proceedings of the Cambridge Philosophical Society  
Zentralblatt

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